Lumped Parameter Modeling of the Ideal Railgun: Examining Maximum Electromechanical Energy Conversion Efficiency

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Student Paper Abstract

Railguns, which accelerate projectiles through electromagnetic repulsion, have been gaining more attention for use in the next generation of all electric naval ships. The military has interest in railguns because they have several unique advantages such as the lack of explosives, extremely long firing ranges, and low cost compared to missiles. Typically, the efficiency of converting electrical energy into kinetic energy is between 10% and 50% for large scale research guns. These railguns operate from a stored energy source, such as a capacitor, in the range of 400 kJ to 32 MJ.

Because of the nature of this technology, many amateur scientists and hobbyists have tried to build their own small scale railguns in the energy range of 100 J to 20 kJ. Each of these amateur railguns, except one, has very poor efficiencies, often below 1%. To explain why only one amateur railgun performed well while the others have poor efficiencies, the author developed a mathematical model of an ideal railgun using lumped parameters in a pair of non-linear second order differential equations. Simulink software was used to solve these equations, and data gathered from tests were compared to the model. Additionally, analytic equations for efficiency were derived.

The result of the modeling shows that the amount of momentum transferred to the projectile is proportional to the action integral, which is the time integral of the current squared. High efficiency is obtained by a circuit that is capable of producing a large amount of action with minimal energy input; this is done directly by minimizing circuit resistance.

To verify the model, a small scale railgun was built using a 5 kJ electrolytic capacitor bank. This railgun was capable of firing solid metal armatures and plasma armatures. Current waveforms were captured using a Rogowski coil transducer and an oscilloscope. The railgun was able to fire a 500mg projectile with approximately 20J of kinetic energy using a plasma armature. However, solid metal armatures could not be fired with significant speed due high friction from metal on metal contact inside the railgun. This low performance was quantitatively predicted by the railgun model since the capacitor bank had a high internal resistance. As a result, an experiment is currently being conducted which will test the model's validity using a low resistance 20 kJ capacitor bank. The 20 kJ railgun has been constructed at Rose-Hulman Institute of Technology and is currently being prepared to fire.

The relationships derived, if proven valid through observation, could be used in the design of amateur railguns to help impove efficiency and performance. These relationships may be extended to improve future large scale railgun designs.

Key Words
Student Paper, Electrical Engineering, Electrical and Computer Engineering

Lumped Parameter Modeling of the Ideal Railgun:
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Introduction

A railgun is an electromechanical device that consists of two parallel conducting rails that are shorted by the armature which can be accelerated to speeds in excess of 3 km/s solely due to magnetic repulsion. Due to their flexibility of application and because they function without chemical explosives, the military has interest in using railguns for defense and combat support. Other potential uses include nuclear fusion, microsatellite launch, and high speed meteor impact study.

The railgun and its power supply form a simple series circuit that can be thought of as a loop of current. The current loop interacts with its own magnetic field to produce a force that acts to expand the loop outwards. By containing the conductors of the loop in a rigid structure while allowing only the armature to move, the current loop, instead of exploding outward, is transformed into a railgun that accelerates the armature outward.

A complete system consists of a power supply, switch, rails, containment structure, and projectile.

Background

The heart of every amateur railgun is its power supply. The power supply must have a high power density. That is, the power supply must be capable of providing a large amount of power in a short time while being cost effective. The most popular solution that meets these requirements is a bank of capacitors. Generally, the main types of capacitors used for railguns are electrolytic and pulsed power capacitors. Electrolytic capacitors are used most frequently because of their availability and comparatively low cost. Energy capacity generally ranges between 100 J and 20 kJ.

The vast majority of amateur railguns has always resulted in poor electromechanical efficiency, typically much less than 1% efficient compared to 10% - 50% efficiency for large scale railguns. One railgun, built by a group of students at the University of Virginia in 2004, was able to achieve 19% efficiency with a 6.5 kJ electrolytic capacitor bank.[1] There has been much debate about how to increase efficiency, and most amateurs tend to agree that the primary way to do this is to increase the peak current flowing through the rails. Amateurs who draw this conclusion tend to wire their capacitors in series to increase the voltage. The rationale behind this conclusion is that the electromagnetic force on the projectile is proportional to the square of the current flowing through the rails, so more current means a lot more force. While this may be a true statement, increasing the peak force alone does not increase the energy transfer to the projectile. In order to efficiently transfer energy to the projectile, it is necessary to

increase the net momentum transfer to the projectile with as little input energy as possible. This means maximizing the net impulse, which is the time integral of force, on the projectile.

Defining the Ideal Simple Railgun

The Lorentz force equation can be applied to a railgun. Assuming the magnetic fields are linear and perpendicular to the flow of current, the force equation is $F = BId$, where B is the magnetic field, I is the current, and d is the width of the armature. If magnetic flux is assumed to be the product of magnetic field and some area, and equal to inductance times current, the force equation can be written as some inductance per unit length times current squared.

All models are simplifications of more complex systems. To simplify the system, unnecessary complexities such as non-linear magnetic materials will not be modeled. To aid the development of a reasonable model, the following simplifications and assumptions are made:

A simple railgun has all of the following qualities:
1. composed of only two parallel conducting non-magnetic rails
2. projectile is solid metal or composed of brushes, i.e. no plasma armatures or arcing
3. projectile starts at rest, i.e. no pre-acceleration injection
4. no iron core

An ideal railgun has all of the following qualities:
1. all energy losses are due to Joule heating and electromechanical energy conversion, e.g. no friction, eddy losses, radiation losses, etc.
2. switch is ideal and lossless
3. circuit resistance is a constant or can be approximated as a weighted average; negligible thermal effects
4. inductance and inductance gradient are constant
5. rails are infinitely long
6. force on the projectile is $F = \frac{1}{2} L'I^2$ for simple railguns where L' is the inductance gradient of the rails and I is the current flowing through them[2]
7. current reaches a steady state value of 0
8. projectile mass is constant

Governing Differential Equations

Using the definition of an ideal simple railgun, the system can be reduced into an RLC circuit as shown in figure 1.
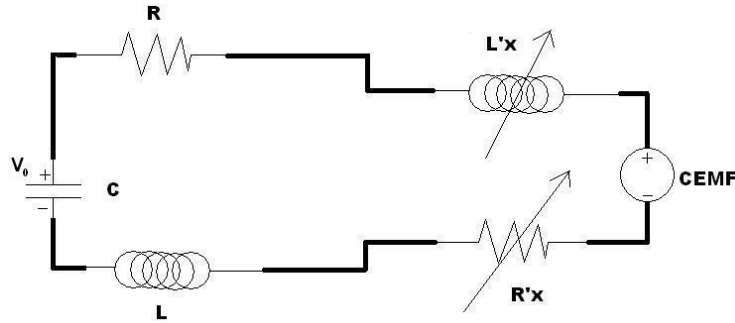
Figure 1: Equivalent RLC circuit of an Ideal Simple Railgun

V_0 is the initial voltage of the capacitor and R' is the resistance gradient of the rails. The symbol x is the position of the armature with mass m. Using Kirchhoff's voltage law, the loop equation shown in figure 2 can be written. The capacitance gradient of the rails, C', is neglected for simplification since the magnitudes of both C' and rail voltage are extremely small in comparison to L' and rail current.

$$\frac{1}{C}\int_0^t i(t)\,dt + i(t)\,R + i(t)\,Rp\,\mathrm{x}(t) + \left(\frac{d}{dt}i(t)\right)L + \left(\frac{d}{dt}i(t)\right)Lp\,\mathrm{x}(t) + Lp\,i(t)\left(\frac{d}{dt}\mathrm{x}(t)\right) = 0$$

| capacitor voltage | Resistive voltage drop | Resistive drop due to increasing resistance | Inductor voltage | Inductor voltage due to increasing inductance | Counter EMF due to Faraday's law |

Figure 2: KVL equation for figure 1.

The counter EMF term is found by using the definition of inductance and Faraday's law. Taking the derivative of the KVL equation with respect to time yields the first differential equation. Since the energy source is the capacitor that is switched on at time = 0, the initial condition for I'(0) is V_0/L.

$$\frac{d^2 I(t)}{dt^2}\cdot\left(L + L'x(t)\right) + \frac{dI(t)}{dt}\cdot\left(R + R'x(t) + 2L'\frac{dx(t)}{dt}\right) + I(t)\cdot\left(\frac{1}{C} + R'\frac{dx(t)}{dt} + L'\frac{d^2 x(t)}{dt^2}\right) = 0$$

(1)

with initial conditions

$$\frac{dI(0)}{dt} = \frac{V_0}{L}, I(0) = 0$$

(2)

Since the system to be modeled is ideal and simple, the only force acting on the projectile is the electromagnetic force, which is #6 on the list of ideal railgun qualities.

$$F = \frac{1}{2}L'I(t)^2$$

(3)

The force equation can be derived using conservation of energy and is standard in the field of electromagnetic launchers. The equations of motion can be easily rewritten to include effects such as friction and air drag, but it is unnecessary for this analysis. Newton's law gives the second differential equation.

$$\frac{d^2 x(t)}{dt^2} = \frac{1}{2m} L' I(t)^2, \quad \frac{dx(0)}{dt} = 0, \quad x(0) = 0 \tag{4}$$

Integrating equation (4) with respect to time gives equations for speed and position, respectively.

$$\frac{dx(t)}{dt} = \int_0^t \frac{1}{2m} L' I(t)^2 dt \tag{5}$$

$$x(t) = \int_0^t \int_0^t \frac{1}{2m} L' I(t)^2 dt dt \tag{6}$$

These equations were solved numerically using MathWork's Simulink software package.

Analytic Equations for Ideal Simple Railguns

The first step is to define the action integral, which is simply the time integral of current squared.

$$A = \int_0^T I(t)^2 dt \tag{7}$$

By integrating the equation for power, $P = I^2 R$, with respect to time, another definition of action is given by

$$A = \frac{E}{R} \tag{8}$$

The energy term, E, in (7) is the total energy dissipated in the resistance R. Equation (7) holds true even when R is time varying; in this case, R would be the total average resistance weighted by current squared.

By integrating the ideal railgun force equation (3) and using the definitions of action, impulse, momentum, and kinetic energy, the expression for kinetic energy is given by

$$KE = \frac{L'^2}{8m} \cdot A^2 \tag{9}$$

Assuming the railgun is ideal, equation (9) states that to increase the kinetic energy of the projectile, it is necessary to either increase inductance gradient, increase action, or decrease projectile mass. Inductance gradient has a typical value of 0.5 µH/m for simple railguns, and it is generally difficult to modify. Assuming the mass of the projectile stays constant once the railgun has been built, the easiest way of increasing kinetic energy is to increase the action integral.

Equation (8) implies that action can be increased by either increasing the energy delivered to the system or by decreasing the resistance. Therefore, to maximize efficiency, it is necessary to provide the most action with minimal input energy, which means minimizing total circuit resistance.

It should be noted that R in equation (8) includes both ohmic resistance, R_T, and an equivalent resistance associated with the conversion of electrical energy to mechanical energy, R_{KE}.

$$R = R_T + R_{KE} \tag{10}$$

This is a direct consequence of conservation of energy. The conversion of energy presents itself as a resistance that actually decreases the amount of action such that it is impossible for even a superconducting circuit to provide enough action to impart more output kinetic energy than input electrical energy. In view of that, equation (9) is more appropriately used either with action known after data has been taken or to predict the minimum amount of action necessary to produce a desired kinetic energy.

By defining efficiency as the ratio of output kinetic energy to input electrical energy, E, the following expression for efficiency is obtained:

$$\eta = \frac{L'^2 \cdot A^2}{8m \cdot E} \tag{11}$$

In order to determine how close to ideal a real railgun performs, equation (12) normalizes the actual result with the predicted result. A normalized efficiency of 1 means the railgun is 100% ideal.

$$normal\ \eta = KE \cdot \frac{8m}{L'^2 \cdot A^2} \tag{12}$$

With equation (10), it is also possible to derive an efficiency expression in terms R_T and input energy instead of action, which takes into account the decrease in action caused by electromechanical energy conversion. This equation is useful for predicting railgun performance before it is constructed and assumes that R' = 0.

$$\eta = \frac{E \cdot L'^2}{E \cdot L'^2 + 4m \cdot R_T^2 + 2R_T \sqrt{4m^2 \cdot R_T^2 + 2m \cdot E \cdot L'^2}} \tag{13}$$

Equation 13 predicts the maximum efficiency possible for an ideal simple railgun.

Experimental Setup

In 2004, a small scale railgun was originally designed for plasma armature projectiles, and it was machined at Ball State University. The plasma armature design was used in order to minimize friction, since it was anticipated that friction would cause problems. The basic design is similar to a musket with a closed breach and loaded from the muzzle. The containment structure is made of fire retardant electrical grade fiberglass with an effective barrel length of 14 inches. The rails are solid ½" square alloy 110 copper bars, and the bore of the railgun is ¼" square. A picture of the railgun is shown in figure 3 with a sample projectile.
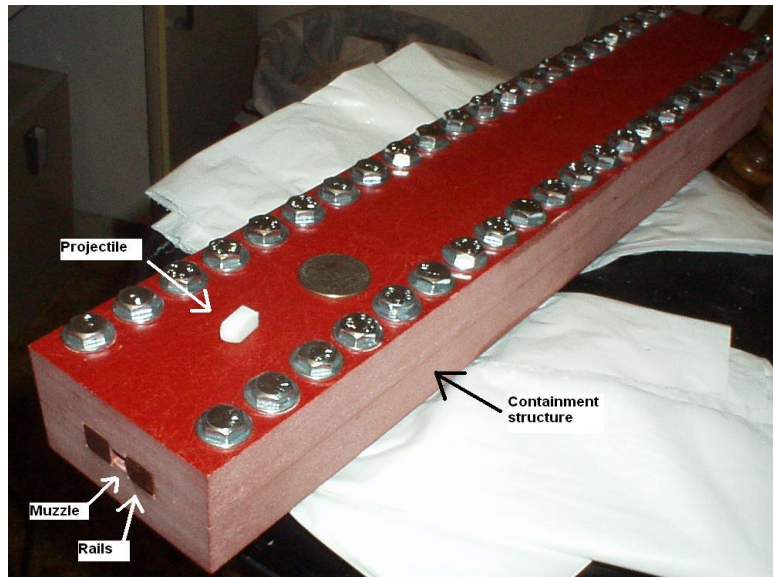

Figure 3: Photo of the railgun with Teflon projectile

The railgun is capable of firing both plasma armature and metal armature projectiles. Plasma armature projectiles are made of Teflon plastic with a small amount steel wool packed behind. When power is switched on, the high currents vaporize the steel wool into plasma that carries the current and pushes the projectile. Because of the gas-like nature of plasma, it is important to completely seal the railgun to prevent leaks which would result in a loss of pressure, hence the musket-like design. Solid armature projectiles were made of brass.

Because of budget constraints, surplus phase control stud mount SCR's were sacrificially used to switch the capacitor bank. The SCR had to be replaced after each shot because they were underrated by orders of magnitude. Mechanical and triggered spark gap switching were not considered because the inherent arcing presents too much loss, and those methods were more complex and expensive to implement.

Budget constraints forced the use of old surplus electrolytic capacitors. A total of 9 capacitors were purchased, each rated for 450V and 6800µF. Of the 9 capacitors, it is possible to wire them in either all in series, all in parallel, or some combination of series/parallel. The parallel configuration was determined to be superior in all aspects

because it minimizes the parasitic resistance of the capacitor bank, which allows it to produce more action than any other configuration. Because the energies are relatively small with respect to the length of the rails, the pulse length of the capacitor discharge was still too short even with all capacitors in parallel.

Data Acquisition

The two most important quantities to measure during testing are current and projectile speed. Referring to figure 1, it was initially thought that, by measuring capacitor bank voltage with an oscilloscope, the current waveform could be inferred using the relationship below.

$$I(t) = C \frac{dV(t)}{dt} \tag{14}$$

However, since the magnitude of current and its derivative is relatively large, the parasitic inductance and resistance of the capacitor bank makes it impossible to accurately measure the voltage across just the capacitance. A better way to measure the current waveform is to use a Rogowski coil, which is a coil of wire with an air core that loops around one conductor. The output of the coil is a signal proportional to the derivative of the current waveform. To form a useful transducer, an analog integrator circuit is used to integrate the output of the coil to give the current waveform which can be captured with an oscilloscope.

Initially, the speed of the projectile was to be measured using an optical chronograph, commonly used to measure the bullet speed of firearms. This proved to be a poor choice since the muzzle flash of the railgun blinds the sensors. The small projectile, typically a ¼" cube, also caused problems with alignment. A novel way to capture time resolved projectile position data is to use a flux ruler, sometimes called a serpentine B-dot probe.[1] A diagram of the flux ruler is shown below in figure 4.
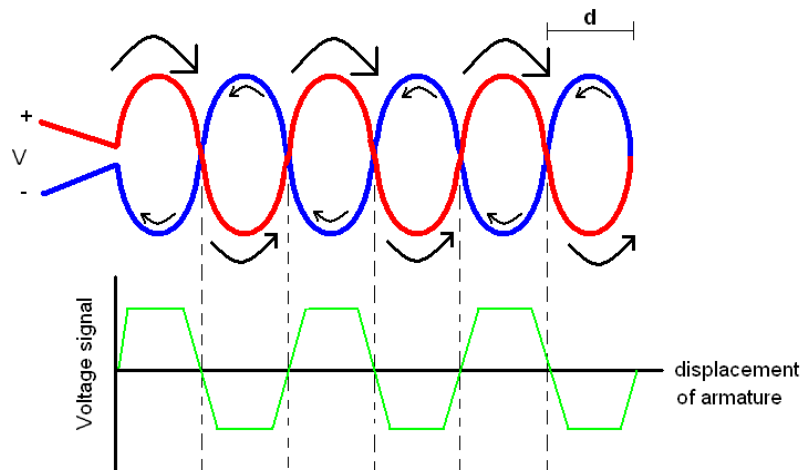

Figure 4: Flux ruler diagram, arrows indicate winding direction

The flux ruler is composed of a serpentine loop of wire that lies on top of the railgun. As the current carrying armature travels down the rails, it induces motional EMF across the terminals of the flux ruler. As the armature approaches a node where the wires cross, the voltage across the terminals drops to zero since the return winding (shown in blue) is 180° out of phase with the red winding (connected to the positive terminal). The times of zero crossings can be used to determine discrete armature position data, which can then be used to find speed as a function of time.

The problem with the flux ruler is that voltage, induced by time varying current, interferes with the desired signal caused by the motional EMF. The magnitude of the interference is often larger than the desired signal. It was also found out by experiment that the distance d, shown on figure 4, must be greater than the length of the projectile in order to induce a measurable motional EMF.

Other methods of measuring speed include high speed camera, ballistic pendulum/sled, and wire breaks. The disadvantages of these methods are complexity, cost, and lack of time resolved data.

Experimental Results

Each trial shot was conducted with the capacitors charged to their rated voltage of 450V. The capacitance of the bank was determined by experiment to be 58 mF, which gives an initial energy storage of 5.87 kJ of energy.

Experimental trial 2 was conducted on February 1, 2006 using a 0.970 g solid brass armature. The friction from the metal on metal contact was so high that the projectile failed to exit the railgun. Post trial inspection revealed that the projectile traveled a distance of about seven inches. No evidence of welding of the projectile to the rails has ever been observed in any trial.

By using a Simulink optimization tool and the captured current waveform as a reference, the parameters of the railgun were estimated. Figure 5 shows the measured response superimposed on the model's predicted response.
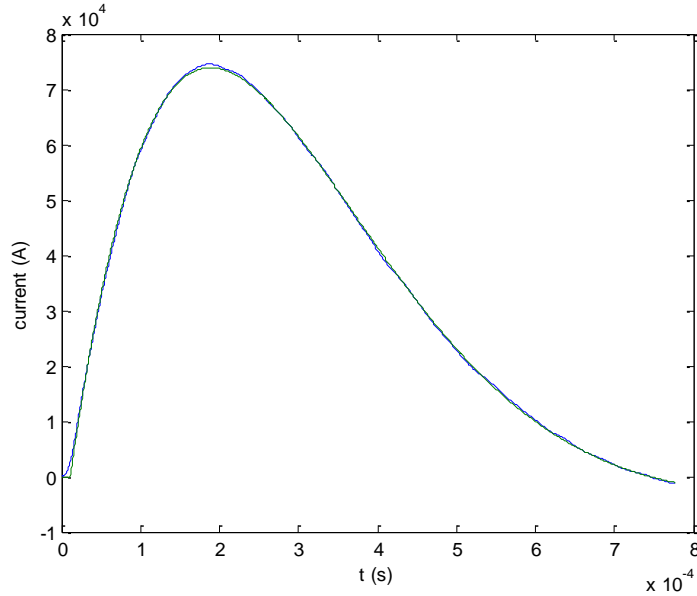
Figure 5: Predicted (green) and measured (blue) current waveforms

Figure 5 shows that the model is able to predict the current waveform very well. The estimated parameters are shown in table 1. The inductance gradient was not tuned by the software because the algorithm could not cope with so many parameters. Instead, trial and error was used to find the best fit value for L'.

Table 1: Table of parameters

R (mΩ)	R' (mΩ/m)	L (µH)	C (F)	L' (µH/m) This value was assumed and not optimized
3.95	0.011	0.446	0.0587	0.3

From the measured current waveform, the action integral for trial 2 is 1.5 MA^2/s. Using equation (9), the predicted kinetic energy assuming no friction is 26 joules, which is a small amount of energy that is easily dissipated by friction. With this amount of action and the initial energy stored, the frictionless efficiency is predicted to be 0.44% while the actual efficiency is 0%.

Figure 6 shows the flux ruler output waveform for trial 3, which used a plasma armature. The plasma armature projectile had significant speed, as evidenced by a large impact crater, but it failed to induce a measurable motional EMF in the flux ruler.
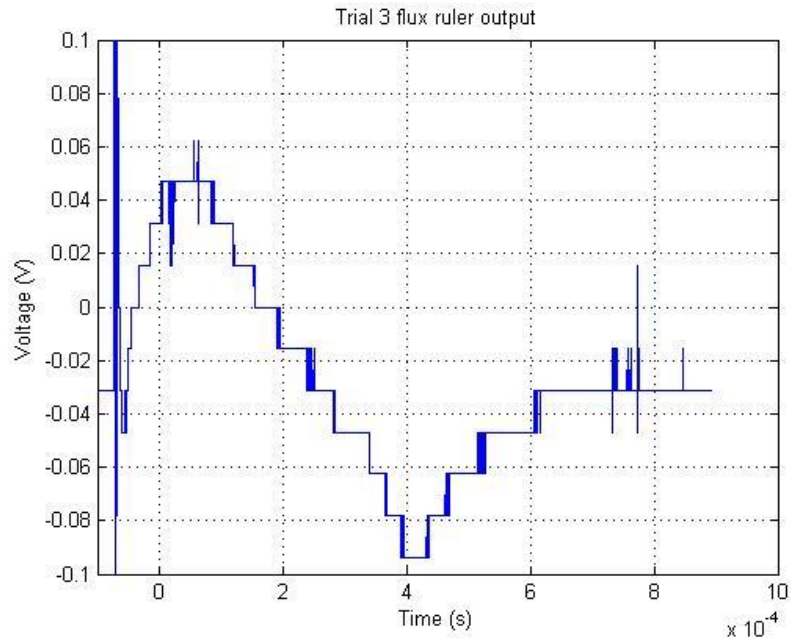
Figure 6: Output of flux ruler for trial 3, note useless output

Trials 1 to 4 used a flux ruler with a spacing of 1 cm, which is about the length of the armature. This configuration leads to a signal with only interference, seen in figure 6, from the time varying current.

Trial 5 was conducted on February 17, 2006 with a 0.834g Teflon plasma armature. The current waveform is shown in figure 7.
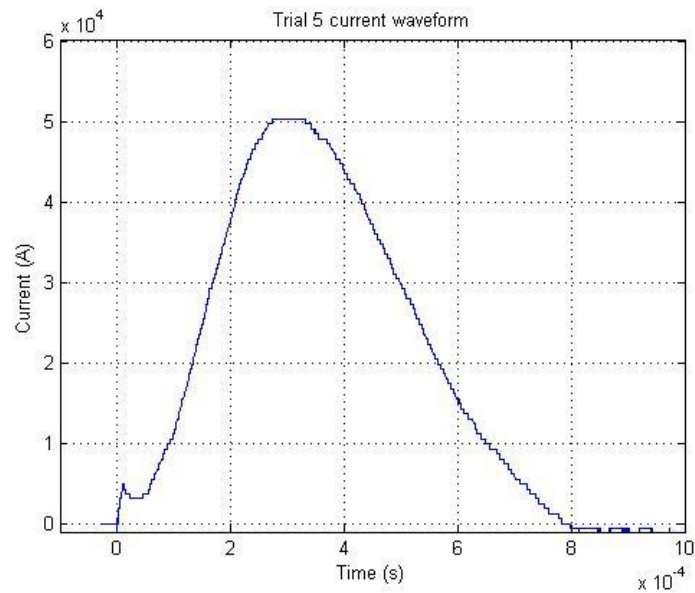

Figure 7: Trial 5 current waveform with plasma armature

Since the plasma armature railgun does not qualify as a simple railgun, it does not follow the previously developed differential equation model. A curious dip in current occurs near the beginning of the waveform. It is thought that the dip is due to the vaporization of the steel wool as it turns into plasma. The resistance of plasma is a function of current and temperature, among other things, so a model could not be developed at this point.

For trial 5, the flux ruler was rewound using a spacing of 2 cm. The output is shown in figure 8.

Figure 8: Trial 5 flux ruler output with desired signal superimposed onto interfering signal

Figure 8 shows an interference signal proportional to the derivative of the current plus the sinusoidal desired signal. The desired signal in figure 8 is the sinusoidal waveform most clearly seen from 0.3 ms to 0.8 ms. Using the time between the peaks of the sinusoidal part and the known loop spacing, the speed of the armature is estimated to be 190 m/s. It is possible to use digital signal processing to filter the signal, but the result is questionable due to uncontrolled phase shifts.

Table 2 shows a summary of the experimental trials where data was collected. Equation (9) is used to find predicted speed and kinetic energy. None of the solid metal projectiles were able to penetrate a 1/16" thick aluminum sheet; plasma armature projectiles could punch a hole through the aluminum and about 300 pages of a phone book.

Table 2: Experimental trial data

Trial #	Date	Armature	Projectile Mass (g)	Action (A^2s)	Pk Current (kA)	Pulse Width (ms)	Actual Speed/ KE (m/s) / (J)	Predicted speed/KE L' assumed 0.3μH/m

								(m/s) / (J)
1	1/30/06	Solid brass	1.906	1.435e6	73.2	0.7	--	113 / 12
2	2/01/06	Solid brass	0.970	1.5e6	74.4	0.7	--	232 / 26
3	2/03/06	Plasma	0.553	7.524e5	52.3	0.7	--	204 / 11.5
4	2/14/06	Solid brass	1.007	1.419e6	69.5	0.7	--	211 / 22
5	2/17/06	Plasma	0.834	7.263e5	50.3	0.8	190 / 15	131 / 7.2

University of Virgina's Railgun

The University of Virginia railgun recorded a 19% efficiency using 6.5 kJ electrolytic capacitor bank wired all in parallel and an augmented railgun, which has an additional set of rails to increase the inductance gradient. The UVA railgun has the following parameters shown in table 3 below:

Table 3: UVA railgun parameters[1]

L' (μH/m) [estimated from graph]	R_T (mΩ) [estimated from graph]	Energy (J)	Projectile mass (g)	Efficiency (%)
0.7	0.5	6500	3.62	19

Equation (13) predicts a maximum efficiency of approximately 25% for these parameters. This result is consistent given the significant effects of friction and other losses.

Conclusions and Recommendations

Since the effects of plasma armatures were not considered in the model, results for plasma armatures do not conform to the model. The unusually shaped current waveforms for plasma armatures were not anticipated. The actual kinetic energy of trial 5 is more than twice the predicted value. However, the absolute difference is 7.8 Joules, which is only a tiny fraction of the initial 5.87 kJ of energy that went into the system. The discrepancy is thought to be due to thermal kinetic energy conversion. As the steel wool explodes into plasma, the current flowing through the plasma generates heat that causes the gasses to expand and push the projectile. This concept is exploited in electrothermal guns where only electricity is used to heat an inert substance to produce pressure to accelerate a projectile.

The action integral for plasma armatures is consistently and significantly lower than the action integral for solid metal armatures, which means that the total average resistance for plasma armatures is higher than for solid armatures. Since the armature type is the only variable that significantly alters action between trials, it can be construed that the plasma itself is highly resistive. Because of the highly resistive nature of the plasma, a large

portion of the initial energy gets dissipated as heat into the plasma, which supports the electrothermal explanation for discrepancy in trial 5.

Based on the information gained from the 5 experimental trials, further investigation is warranted for solid armatures. The low predicted energies for solid armatures are consistent with the observed low speed impacts caused by high metal on metal friction. In order to produce useful results, the capacitor bank needs to be able to deliver more action to overcome friction. With the information from trial 2, the resistance R_T of 3.95 mΩ predicts an efficiency of approximately 0.44%; if the resistance could be reduced to 1 mΩ, the predicted efficiency, from equation (13), is 6%. For 0.5 mΩ, the efficiency would be 18%.

Future Work

For future work, the railgun has been modified to facilitate the use of solid metal armatures. Current waveforms are to be captured using the Rogowski coil transducer as before, and the speed of the projectile will be recorded with a high speed camera. A new capacitor bank, composed of 70 electrolytic capacitors wired in parallel was constructed. Each capacitor is rated for 400 V, 3900 μF, and the nominal ESR is 40 mΩ based on the datasheet value. Table 4 shows the estimated parameters for the new railgun.

Table 4: New railgun parameters

L' (μH/m)	R_T (mΩ)	Energy (J)	Projectile mass (g)	Predicted ideal efficiency [equation (13)] (%)
0.3	0.6	21800	0.6	40

The predicted efficiency of 40% is valid only if the capacitor bank can deliver the 22 kJ before the projectile exits the railgun. The differential equation based simulations indicate that the projectile will exit the railgun well before the capacitor discharges completely, in which case the predicted efficiency is 18%. Effects such as friction, eddy currents, increasing rail resistance, and other factors will lower this number considerably.

The premature exit of the projectile is expected to cause a large muzzle flash due to energy stored in the inductance of the circuit. As a result, a Faraday cage is currently being constructed to protect equipment from electromagnetic radiation. The railgun is expected to fire in Spring of 2008.

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